Solutions to short-answer questions

1 **a** When $t = 0, x = -5$.

5 cm to the left of O

b $$\begin{aligned} \text{When } t = 3, x &= 3^2 - 4 \times 3 - 5 \\ &= -8 \end{aligned}$$

8 cm to the left of O

c $$\begin{aligned} v &= \frac{dx}{dt} \\ &= 2t - 4 \end{aligned}$$

When $t = 0$, -4 cm/s

d $$\begin{aligned} v = 0 \text{ when } 2t - 4 &= 0 \\ t &= 2 \end{aligned}$$

$$\begin{aligned} \text{When } t = 2, x &= 2^2 - 4 \times 2 - 5 \\ &= -9 \end{aligned}$$

At 2 s, 9 cm to the left of O

e $$\begin{aligned} \text{Average velocity} &= \frac{\text{change in position}}{\text{change in time}} \\ &= \frac{-8 - (-5)}{3} = -1 \text{ cm/s} \end{aligned}$$

1 cm/s to the left

f Distance travelled = distance from $t = 0$ to $t = 2$ (when $v = 0$), plus distance from $t = 2$ to $t = 3$

$$\begin{aligned} \text{So distance travelled} &= 4 + 1 \\ &= 5 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Average speed} &= \frac{\text{distance travelled}}{\text{change in time}} \\ &= \frac{5}{3} = 1\frac{2}{3} \text{ cm/s} \end{aligned}$$

(Note: Average velocity has a direction and hence a sign, but average speed does not.)

2 **a** $$\begin{aligned} v &= \frac{dx}{dt} \\ &= 3t^2 - 4t \end{aligned}$$

$$a = \frac{dv}{dt} = 6t - 4$$

When $t = 0, x = 8, v = 0$ and $a = -4$.

8 cm to the right of O, stationary and accelerating at 4 cm/s^2 to the left.

b $v = 0$ when

$$3t^2 - 4t = 0t(3t - 4) \qquad = 0$$

$$t = 0 \text{ or } \frac{4}{3}$$

$t = 0 : x = 8$ and $a = -4$

So 8 cm to the right, -4 cm/s^2

$$t = \frac{4}{3} : x = \frac{64}{27} - \frac{32}{9} + 8 = 6\frac{22}{27}$$

$$a = 8 - 4 = 4$$

So $6\frac{22}{27}$ cm to the right, 4 cm/s^2

3 a Solve $-2t^3 + 3t^2 + 12t + 7 = 0$

Using factors of 7, $t = -1$ gives

$-2 \times (-1)^3 + 3 \times (-1)^2 + 12 \times -1 + 7 = 0$

Dividing by $(t+1)$,

$$\begin{aligned} & -2t^3 + 3tg^2 + 12t + 7 \\ & = -(t+1)(2t^2 - 5t - 7) \\ & = -(t+1)(t+1)(2t-7) \\ & = 0 \end{aligned}$$

$t = 3.5$, as $t = -1$ is usually discarded.

$$\begin{aligned} v &= \frac{dx}{dt} \\ &= -6t^2 + 6t + 12 \\ a &= \frac{dv}{dt} = -12t + 6 \end{aligned}$$

When $t = 3.5$

$$\begin{aligned} v &= -6 \times 3.5^2 + 6 \times 3.5 + 12 \\ &= -40.5 \text{ cm/s} \\ a &= \frac{dv}{dt} \\ &= -12 \times 3.5 + 6 \\ &= -36 \text{ cm/s}^2 \end{aligned}$$

b

$$\begin{aligned} v &= 0 \\ -6t^2 + 6t + 12 &= 0 \\ t^2 - t - 2 &= 0 \\ (t+1)(t-2) &= 0 \\ t &= 2 \end{aligned}$$

After 2s (discarding $t = -1$)

c Distance travelled in first 2 seconds

$$\begin{aligned} & = (-2 \times 2^3 + 3 \times 2^2 + 12 \times 2 + 7) \\ & \quad - (-0 + 0 + 0 + 7) \\ & = 20 \text{ cm} \end{aligned}$$

Distance travelled from $t = 2$ to $t = 3$ is

$$\begin{aligned} & |(-2 \times 3^2 + 3 \times 32 + 12 \times 3 + 7) \\ & \quad - (-2 \times 2^3 + 3 \times 2^2 + 12 \times 2 + 7)| \\ & = |16 - 27| \\ & = 11 \text{ cm} \end{aligned}$$

Distance travelled in first 3 s

$$\begin{aligned} & = 20 + 11 \\ & = 31 \text{ cm} \end{aligned}$$

4 a i

$$\begin{aligned} x_1\left(\frac{1}{2}\right) &= \left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 \\ &= \frac{1}{8} - \frac{1}{4} \\ &= -\frac{1}{8} \end{aligned}$$

$\frac{1}{8}$ cm to the left

ii $$a_1(t) = \frac{d^2x}{dt^2} = 6t - 2$$

$$a_1\left(\frac{1}{2}\right) = 6 \times \frac{1}{2} - 2 = 1 \text{ cm/s}^2$$

iii $$v_2(t) = \frac{dx}{dt} = 2t$$

$$v_2 = 2 \times \frac{1}{2} = 1 \text{ cm/s}$$

b i

$$x_1(t) = x_2(t)$$
$$t^3 - t^2 = t^2$$
$$t^3 - 2t^2 = 0$$
$$t^2(t-2) = 0$$
$$t = 0 \text{ and } 2$$

The particles will have the same position at the start and after 28 s.

ii Let the distance between the particles be $y = |t^3 - 2t^2|$.

Define $y = t^3 - 2t^2$:

$$\frac{dy}{dt} = 3t^2 - 4t = t(3t-4) = 0 \text{ when } t = 0 \text{ and } \frac{4}{3}$$

When $t = 0, y = 0$.

$$\text{When } t = \frac{4}{3}, y = \frac{64}{27} - \frac{32}{9} = -1\frac{5}{27}$$

$$\text{When } t = 2, y = 8 - 2 \times 4 = 0$$

The maximum distance the particles are apart in the first 2 s is $\frac{32}{27} = 1\frac{5}{27}$ cm

5 a

$$a = 6t$$
$$v = 3t^2 + c$$

When $t = 0, v = 0$.

$$0 = 0 + c$$
$$c = 0$$
$$\therefore \quad v = 3t^2$$

$$\text{When } t = 2, v = 3 \times 4 = 12 \text{ m/s}$$

b

$$v = 3t^2$$
$$x = t^3 + d$$

When $t = 0, x = 0$.

$$0 = 0 + d$$
$$d = 0$$
$$x = t^3$$

Since the particle starts at the origin, its displacement is $s = x = t^3$.

6 a $a = 3 - 2t$

$v = 3t^2 - t^2 + c$

When $t = 0, v = 4$.

$4 = 0 - 0 + c$

$c = 4$

$$\begin{aligned} v = 3t - t^2 + 4 &= 0 \\ -(t^2 - 3t - 4) &= 0 \\ -(t-4)(t+1) &= 0 \\ t &= 4 \end{aligned}$$

After 48 s

b $v = 3t - t^2 + 4$

$x = \frac{3t^2}{2} - \frac{t^3}{3} + 4t + d$

When $t = 0, x = 0$.

$0 = 0 - 0 + 0 + d$

$d = 0$

$x = \frac{3t^2}{2} - \frac{t^3}{3} + 4t$

$$\begin{aligned} \text{When } t = 4, x &= \frac{3 \times 4^2}{2} - \frac{4^3}{3} \\ &\quad + 4 \times 4 \\ &= 18\frac{2}{3} \end{aligned}$$

$18\frac{2}{3}$ m to the right

c $$\begin{aligned} \text{When } t = 4, a &= 3 - 2 \times 4 \\ &= -5 \text{ m/s}^2 \end{aligned}$$

d $a = 3 - 2t = 0$

$t = 1.5$ s

e When $t = 1.5$,

$$\begin{aligned} v &= 3t - t^2 + 4 \\ &= 3 \times 1.5 - 1.5^2 + 4 \\ &= 6.25 \text{ m/s} \end{aligned}$$

7 a $s = \frac{2t^3}{3} - \frac{3t^4}{4} + c$

When $t = 0, s = 0$.

$0 = 0 - 0 + c$

$c = 0$

$s = \frac{2t^3}{3} - \frac{3t^4}{4}$

$$\begin{aligned} \text{When } t = 1, x &= \frac{2 \times 1^3}{3} - \frac{3 \times 1^4}{4} \\ &= \frac{2}{3} - \frac{3}{4} = \frac{1}{12} \end{aligned}$$

$\frac{1}{12}$ m to the left.

b When $t = 1, v = 2 - 3$

$$= -1 \text{ m/s}$$

c $a = \frac{dv}{dt}$

$$= 4t - 9t^2$$

When $t = 1, a = 4 \times 1 - 9 \times 1^2$

$$= -5 \text{ m/s}^2$$

8 a $v = \frac{1}{2t^2} = \frac{1}{2}t^{-2}$

$$a = \frac{dv}{dt}$$

$$= \frac{1}{2} \times (-2t^{-3}) = -\frac{1}{t^3}$$

b $v = \frac{1}{2}t^{-2}$

$$s = -\frac{1}{2}t^{-1} + c$$

When $t = 1, s = 0$.

$$0 = -\frac{1}{2} \times 1^{-1} + c$$

$$0 = -\frac{1}{2} + c$$

$$c = \frac{1}{2}$$

$$s = \frac{1}{2} - \frac{1}{2t}$$

9 a $a = \frac{dv}{dt}$

$$= 3t^2 - 22t + 24$$

b Solve for $v = 0$.

$t^3 - 11t + 24t = t(t - 3)(t - 8)$

Since motion is only defined for $t \geq 0$, it cannot be said to change direction at $t = 0$.

$\therefore \quad t = 3$

$$a = 3 \times 3^2 - 22 \times 3 + 24$$

$$= -15 \text{ m/s}^2$$

c $v = t^3 - 11t^2 + 24t$

$$x = \frac{t^4}{4} - \frac{11t^3}{3} + 12t^2 + c$$

When $t = 0, x = 0$

$$0 = 0 - 0 + 0 + c$$

$$c = 0$$

$$x = \frac{t^4}{4} - \frac{11t^3}{3} + 12t^2$$

When $t = 5, x = \frac{5^4}{4} - \frac{11 \times 5^3}{3}$

$$+ 12 \times 5^2$$

$$= -2\frac{1}{2}$$

When $t = 3, x = \frac{3^4}{4} - \frac{11 \times 3^3}{3}$

$$+12 \times 3^2$$
$$= 29\frac{1}{4}$$

When $t = 0, x = 0$.

Total distance

$$= 29\frac{1}{4} + \left(29\frac{1}{4} + 2\frac{1}{12}\right)$$
$$= 60\frac{7}{12} \text{ m}$$

$2\frac{1}{12}$ m left of O, 60 $\frac{7}{12}$ m

10 $u = 20, v = 0, t = 4$

$$s = \frac{1}{2}(u+v)t$$
$$= \frac{1}{2} \times 20 \times 4$$
$$= 40 \text{ m}$$

11a $u = 0, v = 30, t = 12$

$$v = u + at$$
$$30 = 12a$$
$$a = \frac{30}{12}$$
$$= 2.5 \text{ m/s}^2$$

b $u = 30, v = 50, a = 2.5$

$$v = u + at$$
$$50 = 30 + 2.5t$$
$$2.5t = 20$$
$$t = 8 \text{ s}$$

c

$$s = ut + \frac{1}{2}at^2$$
$$= 0 + \frac{1}{2} \times 2.5 \times 20^2$$
$$= 500 \text{ m}$$

d

$$100 \text{ km/h} = 100 \div 3.6$$
$$= \frac{250}{9} \text{ m/s}$$

$u = 0, v = \frac{250}{9}, a = 2.5$

$$v = u + at$$
$$\frac{250}{9} = 2.5t$$
$$t = \frac{250}{9 \times 2.5}$$
$$= 11\frac{1}{9} \text{ s}$$

12a

$$100 \text{ km/h} = 100 \div 3.6$$
$$= \frac{50}{3} \text{ m/s}$$

$u = 0, v = \frac{50}{3}, a = 0.4$

$$v = u + at$$
$$\frac{50}{3} = 0.4t$$
$$t = \frac{50}{3 \times 0.4}$$
$$= 41\frac{2}{3} \text{ s}$$

b $$s = \frac{1}{2}(u + v)t$$
$$= \frac{1}{2} \times \frac{50}{3} \times \frac{125}{3}$$
$$= 347\frac{2}{9} \text{ m}$$

13a $$u = 35, s = 0, a = -9.8$$
$$s = ut + \frac{1}{2}at^2$$
$$0 = 3.5t - 4.9t^2$$
$$0.7t(50 - 7t) = 0$$
$$t = \frac{50}{7} = 7\frac{1}{7} \text{ s}$$
$$\approx 7.143 \text{ s}$$

b $$u = 35, s = 60, a = -9.8$$
$$s = ut + \frac{1}{2}at^2$$
$$60 = 35t - 49t^2$$
$$4.9t^2 - 35t + 60 = 0$$
$$49t - 250t + 600 = 0$$
$$(7t - 20)(7t - 30) = 0$$
$$t = 2\frac{6}{7} \text{ or } 4\frac{2}{7}$$

After $2\ \frac{6}{7}$ s (going up) and $4\ \frac{2}{7}$ s (going down)

14a Maximum height occurs when $v = 0$.
$$u = 19.6, a = -9.8, v = 0$$
$$v = u + at$$
$$0 = 19.6 - 9.8t$$
$$t = \frac{19.6}{9.8} = 2 \text{ s}$$

b $$s = ut + \frac{1}{2}at^2$$
$$= 19.6 \times 2 + \frac{1}{2} \times -9.8 \times 2^2$$
$$= 19.6 \text{ m}$$

With respect to ground level, height $= 19.6 + 20 = 39.6 \text{ m}$

c $$u = 19.6, s = 0, a = -9.8$$
$$s = ut + \frac{1}{2}at^2$$
$$0 = 19.6t + \frac{1}{2} \times -9.8 \times t^2$$
$$0 = 19.6t - 4.9t^2$$
$$0 = 4.9t(4 - t)$$
$$t = 4 \text{ s}$$

d $u = 19.6, s = -20, a = -9.8$

$$s = ut + \frac{1}{2}at^2$$

$$-20 = 19.6t + \frac{1}{2} \times -9.8 \times t^2$$

$$-20 = 19.6t - 4.9t^2$$

$$4.9t^2 - 19.t - 20 = 0$$

$$49t^2 - 196t - 200 = 0$$

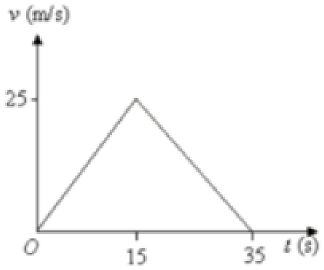
$$\begin{aligned} \triangle &= b^2 - 4ac \\ &= 196^2 - 4 \times 49 \times -200 \\ &= 77\,616 \end{aligned}$$

$$\sqrt{\triangle} \approx 278.596$$

Since the discriminant is irrational, solve using the quadratic formula:

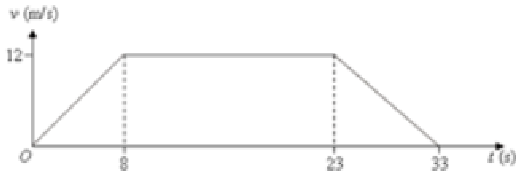
$$\begin{aligned} t &= \frac{196 \pm 278.596}{98} \\ &\approx 4.84 \text{ or } -0.84 \\ &\approx 4.84 \text{ s (since } t > 0) \end{aligned}$$

15



$$\begin{aligned} \text{Distance} &= \text{area} \\ &= \frac{1}{2} \times 35 \times 25 \\ &= 437.5 \text{ m} \end{aligned}$$

16



a

$$\begin{aligned} \text{Distance} &= \text{trapezium area} \\ &= \frac{1}{2} \times (33 + 15) \times 12 \\ &= 288 \text{ m} \end{aligned}$$

b Halfway point is 1448 m.

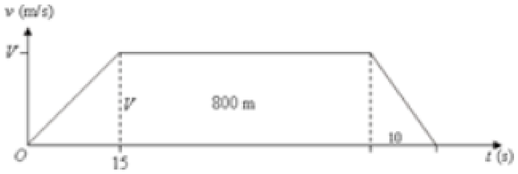
The car has travelled $\frac{1}{2} \times 8 \times 12 = 48$ m in the first 88 s.

It must travel $144 - 48 = 96$ m at 12 m/s.

This will take $96 \div 12 = 8$ s.

Total of 16 s.

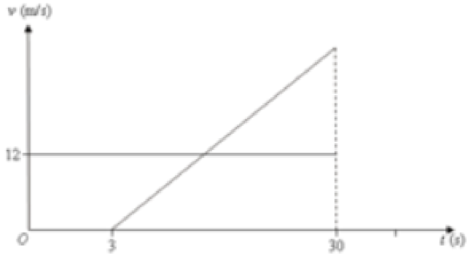
17



Since the vehicle travels $1 \text{ km} = 1000 \text{ m}$, adding the two triangles together should give an area equal to a distance of 2008 m. The triangles have a combined base of 25.

$$\begin{aligned} A &= \frac{1}{2} \times 25 \times V \\ &= 200 \\ V &= \frac{200 \times 2}{25} \\ &= 16 \text{ m/s} \end{aligned}$$

18 After 3 s, the first car has travelled $12 \times 3 = 36 \text{ m}$.



Let the second car's final velocity be $V \text{ m/s}$. The two areas will be equal.

$$\begin{aligned} \frac{1}{2} \times 27 \times V &= 12 \times 30 \\ &= 360 \\ V &= \frac{2 \times 360}{27} \\ &= \frac{80}{3} \end{aligned}$$

For constant acceleration,

$$\begin{aligned} \text{acceleration} &= \frac{\text{change in velocity}}{\text{change in time}} \\ &= \frac{80}{3 \times 27} = \frac{80}{81} \text{ m/s}^2 \end{aligned}$$

19a
$$\begin{aligned} v &= \frac{10^2}{4} - 3 \times 10 + 5 \\ &= 0 \text{ m/s} \end{aligned}$$

b
$$\begin{aligned} a &= \frac{dv}{dt} \\ &= \frac{2t}{4} - 3 \\ &= \frac{t}{2} - 3 \end{aligned}$$

When $t = 0$, $a = -3 \text{ m/s}^2$.

c Minimum velocity occurs when $a = 0$.

$$\begin{aligned} \frac{t}{2} - 3 &= 0 \\ t &= 6 \end{aligned}$$

When $t = 6$,

$$v = \frac{6^2}{4} - 3 \times 6 + 5$$
$$= -4 \text{ m/s}$$

d $$v = \frac{t^2}{4} - 3t + 5$$
$$x = \frac{t^3}{12} - \frac{3t^2}{2} + 5t + c$$

When $t = 0, x = 0$.

$$0 = 0 - 0 + 0 + c$$
$$c = 0$$
$$x = \frac{t^3}{12} - \frac{3t^2}{2} + 5t$$

Check for change of direction of velocity.

$$v = 0 \text{ if } \frac{t^2}{4} - 3t + 5 = 0$$
$$t^2 - 12t + 20 = 0$$
$$(t-2)(t-10) = 0$$
$$t = 2 \text{ or } 10$$

There will be no change of direction of velocity in the first 2s.

When $t = 2$,

$$x = \frac{2^3}{12} - \frac{3 \times 2^2}{2} + 5 \times 2$$
$$= \frac{2}{3} - 6 + 10$$
$$= 4\frac{2}{3} \text{ m}$$

e When $t = 3$,

$$x = \frac{3^3}{12} - \frac{3 \times 3^3}{2} + 5 \times 3$$
$$= \frac{9}{4} - \frac{27}{2} + 15$$
$$= 3\frac{3}{4} \text{ m}$$

Distance travelled in the third second

$$= 4\frac{2}{3} - 3\frac{3}{4}$$
$$= \frac{11}{12} \text{ m (to the left)}$$

20a $$a = 2 - 2t$$
$$v = 2t = t^2 + c$$

When $t = 3, v = 5$.

$$5 = 2 \times 3 - 3^2 + c$$
$$5 = -3 + c$$
$$c = 8$$
$$v = 2t - t^2 + 8$$

b $$v = 2t - t^2 + 8$$
$$x = t^2 - \frac{t^3}{3} + 8t + d$$

When $t = 0, x = 0$.

$$0 = 0 - 0 + 0 + d$$
$$d = 0$$
$$x = t^2 - \frac{t^3}{3} + 8t$$

21a $a = 4 - 4t$

$v = 4t - 2t^2 + c$

When $t = 0, v = 6$.

$6 = 0 - 0 + c$

$c = 6$

$v = 4t - 2t^2 + 6$

$= 6 + 4t - 2t^2$

b Minimum velocity occurs when $a = 0$.

i $4 - 4t = 0$

$t = 1$

$v = 6 + 4t - 2t^2$

$= 6 + 4 \times 1 - 2 \times 1^2$

$= 8 \text{ m/s}$

ii $6 + 4t - 2t^2 = 6$

$4t - 2t^2 = 0$

$2t(2 - t) = 0$

So the velocity of P is again 6 m/s after 28 s.

iii $6 + 4t - 2t^2 = 0$

$-2t^2 + 4t + 6 = 0$

$t^2 - 2t - 3 = 0$

$(t - 3)(t + 1) = 0$

$t = 3$

$x = -\frac{2t^3}{3} + 2t^2 + 6t + d$

$x = 0 \text{ when } t = 0$

$\therefore d = 0$

$x = -\frac{2t^3}{3} + 2t^2 + 6t$

When $t = 3$,

$x = -\frac{2 \times 3^3}{3} + 2 \times 3^2 + 6 \times 3$

$= 18 \text{ m}$

22a When $t = 0, a = 27 \text{ m/s}^2$.

b $a = 27 - 4t^2$

$v = 27t - \frac{4t^3}{3} + c$

When $t = 0, v = 5$.

$5 = 0 - 0 + c$

$c = 5$

$v = 27t - \frac{4t^3}{3} + 5$

$\text{When } t = 3, v = 27 \times 3 - \frac{4 \times 3^3}{3} + 5$

$= 50 \text{ m/s}$

c $$v = 27t - \frac{4t^3}{3} + 5 = 5$$

$$27t - \frac{4t^3}{3} = 0$$
$$81t - 4t^3 = 0$$
$$t(81 - 4t^2) = 0$$
$$t(9 - 2t)(9 + 2t) = 0$$
$$t = 4.5 \text{ s}$$

23a $a = 3 - 3t$

$v = 3t = \frac{3t^2}{2} + c$

When $t = 0, v = 2.$

$2 = 0 - 0 + c$

$c = 2$

$v = 3t - \frac{3t^2}{2} + 2$

When $t = 4, v = 3 \times 4 - \frac{3 \times 4^2}{2} + 2$

$= -10 \text{ m/s}$

b $v = 3t - \frac{3t^2}{2} + 2$

$x = \frac{3t^2}{2} - \frac{t^3}{2} + 2t + d$

When $t = 0, x = 0.$

$0 = 0 - 0 + 0 + d$

$d = 0$

$x = \frac{3t^2}{2} - \frac{t^3}{2} + 2t$

When $t = 4, x = \frac{3 \times 4^2}{2} - \frac{4^3}{2} + 24$

$= 24 - 32 + 8$

$= 0$

24a $t^2 - 10t + 24 = 0$

$(t - 4)(t - 6) = 0$

$t = 4 \text{ and } 6$

b $v = t^2 - 10t + 24$

$x = \frac{t^3}{5} - 5t^2 + 24t + c$

When $t = 0, x = 0.$

$0 = 0 - 0 + 0 + c$

$c = 0$

$x = \frac{t^3}{3} - 5t^2 + 24t$

When $t = 3, x = \frac{3^3}{3} - 5 \times 3^2$

$+ 24 \times 3$

$= 36 \text{ m}$

c $a = 2t - 10 < 0$

$2t < 10$

$t < 5$

Since $t \geq 0, 0 \leq t < 5$

Solutions to multiple-choice questions

1 **A** When $t = 0, x = 0$

2 **E** When $t = 0, x = 0$.

$$\begin{aligned}\text{When } t = 2, x &= -2^3 + 7 \times 2^2 - 12 \times 2 \\ &= -4\end{aligned}$$

$$\begin{aligned}\text{Average velocity} &= \frac{\text{change in position}}{\text{change in time}} \\ &= -\frac{4}{2} \\ &= -2 \text{ cm/s}\end{aligned}$$

3 **C** $v = 4t - 3t^2 + c$

When $t = 0, v = -1$

$$\begin{aligned}-1 &= 0 - 0 + c \\ c &= -1 \\ v &= 4t - 3t^2 - 1\end{aligned}$$

$$\begin{aligned}\text{When } t = 1, v &= 4 \times 1 - 3 \times 1^2 - 1 \\ &= 0 \text{ m/s}\end{aligned}$$

4 **C** $u = 0, s = 90, a = 1.8$

$$\begin{aligned}s &= ut + \frac{1}{2}at^2 \\ 90 &= \frac{1}{2} \times 1.8 \times t^2 \\ 90 &= 0.9t^2 \\ t^2 &= 100 \\ t &= 10 \text{ s}\end{aligned}$$

5 **E** $60 \text{ km/h} = 60 \div 3.6$

$$= \frac{50}{3} \text{ m/s}$$

$$u = 0, v = \frac{50}{3}, t = 4$$

$$\begin{aligned}v &= u + at \\ \frac{50}{3} &= 4a \\ a &= \frac{50}{12} = \frac{25}{6} \text{ m/s}^2\end{aligned}$$

6 **C** $60 \text{ km/h} = 60 \div 3.6$

$$= \frac{50}{3} \text{ m/s}$$

$$u = 0, v = \frac{50}{3}, t = 4$$

$$s = \frac{1}{2}(u + v)t$$

$$= \frac{1}{2} \times \frac{50}{3} \times 4$$
$$= \frac{100}{3}\text{ m}$$

7 **D** Distance

$$= \text{area under graph}$$
$$= \text{triangle} + \text{trapezium} + \text{triangle}$$
$$= \frac{1}{2} \times 4 \times 10 + \frac{1}{2} \times (10 + 25) \times 2 + \frac{1}{2} \times 9 \times 25$$
$$= 20 + 25 + 112.5$$
$$= 167.5\text{ m}$$

8 **E** $u = 0, a = 9.8, s = 40$

$$v^2 = u^2 + 2as$$
$$= 0 + 2 \times 9.8 \times 40$$
$$= 784$$
$$v = \sqrt{784} = 28\text{ m/s}$$

9 **A** $u = 20, v = 0, a = -4$

$$v = u + at$$
$$0 = 20 - 4t$$
$$t = 5$$
$$s = \frac{1}{2}(u + v)t$$
$$= \frac{1}{2} \times 20 \times 5$$
$$= 50\text{ m}$$

10 **D** $v = 6t^2 - 5t + c$

When $t = 0, v = 1$.

$$1 = 0 - 0 + c$$
$$c = 1$$
$$v = 6t^2 - 5t + 1$$

When $t = 1, v = 6 \times 1^2 - 5 \times 1 + 1$
$$= 2\text{ m/s}$$

Solutions to extended-response questions

1 **a** When $t = 0, x = -\frac{7}{3}$

Initial displacement is $\frac{7}{3}$ cm to the left of O.

b $v = t^2 - 4t + 4$

When $t = 0, v = 4$

Initial velocity is 4 cm/s.

c $a = 2t - 4$

When $t = 3, a = 2(3) - 4 = 2$

Acceleration after three seconds is 2 cm/s^2.

d When $v = 0$,

$$t^2 - 4t + 4 = 0$$
$$\therefore (t - 2)^2 = 0$$

$\therefore t = 2$

Velocity is zero after two seconds.

e When $v = 0, t = 2$

$$\therefore x = \frac{1}{3}(2)^3 - 2(2)^2 + 4(2) - \frac{7}{3}$$
$$= \frac{8}{3} - 8 + 8 - \frac{7}{3}$$
$$= \frac{1}{3}$$

When the velocity is zero, the particle is $\frac{1}{3}$ cm to the right of O.

f When $x = 0, \frac{1}{3}t^3 - 2t^2 + 4t - \frac{7}{3} = 0$

Try $t = 1$

$$\text{LHS} = \frac{1}{3}(1)^3 - 2(1)^2 + 4(1) - \frac{7}{3}$$
$$= \frac{1}{3} - 2 + 4 - \frac{7}{3}$$
$$= 0$$

$\therefore \therefore \text{LHS} = \text{ RHS and } t = 1$

The displacement is zero after one second.

Also $3P(t) = t^3 - 6t^2 + 12t - 7 = (t-1)(t^2 - 5t + 7)$
and $t^2 - 5t + 7$ is irreducible since $\Delta = 25 - 4 \times 7 < 0$

2 a

$$x = t^4 + 2t^2 - 8t$$
$$v = \frac{dx}{dt}$$
$$= 4t^3 + 4t - 8$$

$\text{When } t = 0, v = -8$

Since the initial velocity is negative, the particle moves first to the left.

b $\text{When } v = 0, 4t^3 + 4t - 8 = 0$ After one second, the particle is instantaneously at rest.

$$4(t^3 + t - 2) = 0$$
$$\therefore 4(t-1)(t^2 + t + 2) = 0$$
$$\therefore t = 1$$
$$\text{For } t > 1, t - 1 > 0 \text{ and } t^2 + t + 2 > 0$$
$$\therefore 4(t-1)(t^2 + t + 2) > 0$$
$$\therefore v > 0$$

Hence at one second the particle has travelled the greatest distance to the left.

c As $v > 0$ when $t > 1$, the particle always moves to the right for $t > 1$.

3 a The rocket crashes when $h = 0$

$$\text{i.e.} 6t^2 - t^3 = 0$$
$$\therefore t^2(6 - t) = 0$$
$$\therefore t = 0 \text{ or } 6$$
$$\therefore t = 6 \text{ since } t = 0 \text{ represents take-off.}$$
$$v = \frac{dv}{dh}$$
$$= 12t - 3t^2$$

$\text{When } t = 6, v = 12(6) - 3(6)^2$

$$= 72 - 108$$
$$= -36$$

The rocket crashes after six seconds with a velocity of -36 m/s.

b When $v = 0, 12t - 3t^2 = 0$

$$\therefore 3t(4-t) = 0$$

$$\therefore t = 0 \text{ or } 4$$

$$\text{When } t = 4,\ h = 6(4)^2 - (4)^3 = 96 - 64 = 32$$

The speed of the rocket is zero at take-off and after four seconds. The maximum height of the rocket is 32 metres after four seconds.

c

$$a = \frac{dv}{dt} = 12 - 6t$$

$$\text{When } a < 0, 12 - 6t < 0$$

$$\therefore 12 < 6t$$

$$\therefore 2 < t$$

The acceleration becomes negative after two seconds.

4

- $x(1) - x(0) = 15,1$
- $x(2) - x(1) = 5.3 \qquad \text{difference } -9.8$
- $x(3) - x(2) = -4.5 \qquad \text{difference } -9.8$
- $x(4) - x(3) = -14.3 \qquad \text{difference } -9.8$
- $x(5) - x(4) = -24.1 \qquad \text{difference } -9.8$
- $x(6) - x(5) = -33.9 \qquad \text{difference } -9.8$
- $x(7) - x(6) = -43.7 \qquad \text{difference } -9.8$
- $x(8) - x(7) = -53.5 \qquad \text{difference } -9.8$
- $x(9) - x(8) = -63.3 \qquad \text{difference } -9.8$
- $x(10) - x(9) = -73.1 \qquad \text{difference } -9.8$

The body has a constant acceleration of -9.8 m/s ^{2} which is the acceleration due to gravity.

5 a Let $a = -g$ (m/s ^{2}), $v = 0$ (m/s)

Using $v = u + at$,

$$t = \frac{v-u}{a} = \frac{0-u}{-g} = \frac{u}{g},$$

as required.

b When $t = \frac{u}{g}, v = 0$

$$s = \frac{1}{2}(u+v)t = \frac{1}{2}(u+0)\frac{u}{g} = \frac{u^2}{2g}$$

The particle will have travelled $\frac{2u^2}{2g} = \frac{u^2}{g}$ metres to return to its point of projection.

Consider the path of the particle from its highest point when its velocity is zero, until it returns to the point of projection $\frac{u^2}{2g}$ downwards.

$$\text{Then } u = 0, s = \frac{u^2}{2g}, a = g$$
$$\text{and } s = ut + \frac{1}{2}at^2$$
$$\therefore = 0 \times t + gt^2$$
$$\therefore \frac{u^2}{2g} = gt^2$$
$$\therefore t^2 = \frac{u^2}{g^2}$$
$$\therefore t = \frac{u}{g} (t = -\frac{u}{g} \quad \text{is discounted as } t > 0)$$

Hence the total time taken is $\frac{u}{g} + \frac{u}{g} = \frac{2u}{g}$ seconds, as required.

c For the return downwards, $u = 0, t = \frac{u}{g}, a = g$

$$v = u + at$$
$$= 0 + g \times \frac{u}{g}$$
$$= u$$

Hence the speed of returning to the point of projection is u m/s.

6 Consider the throw of the stone to its maximum height. $u = 14, a = 9.8, v = 0$

$$t = \frac{v-u}{a}$$
$$= \frac{0-14}{-9.8}$$
$$= \frac{10}{7}$$

It therefore takes $2 \times \frac{10}{7} = \frac{20}{7}$ seconds for the stone to reach the top of the mine shaft on its descent.

From this point,

$u = -14, a = -9.8, s = ut + \frac{1}{2}at^2$

$\therefore s = -14t - 4.9t^2 \ldots (1)$

When the stone reaches the top of the mine shaft, the lift has been descending for $\frac{20}{7} + 5 = \frac{55}{7}$ seconds and has travelled $\frac{55}{7} \times 3.5 = 27.5$ metres.

From this point,

$s = -27.5 - 3.5t \qquad (\text{ for the lift}) \ldots (2)$

Equating (1) and (2) to find the point of impact.

$$-14t - 4.9t^2 = -27.5 - 3.5t$$
$$\therefore 4.9t^2 + 10.5t - 27.5 = 0$$
$$\therefore t = \frac{-10.5 \pm \sqrt{10.5^2 - 4 \times 4.9 \times (-27.5)}}{2 \times 4.9}$$
$$= 1.42857\ldots$$

(the negative solution is not practical)

$$\text{When } t = 1.42857\ldots,$$
$$s = -27.5 - 3.5 \times 1.42857\ldots$$
$$= -32.85013\ldots$$

Hence the depth of the lift when the stone hits it is 33 metres, to the nearest metre.

7 a

$$90 \text{ km/h} = 90 \times \frac{5}{18} \text{ m/s}$$
$$= 25 \text{ m/s}$$
$$v = -\frac{25}{5}t + 25$$
$$\therefore v = -5t + 25, \qquad 0 \le t \le 5$$

b

$$\text{Distance travelled} = \text{area under the graph}$$
$$= \frac{1}{2} \times 25 \times 5$$
$$= 62.5$$

The distance travelled in five seconds is 62.5 metres.

8

$$x = 3t^4 - 4t^3 + 24t^2 - 48t$$
$$v = \frac{dx}{dt}$$
$$= 12t^3 - 12t^2 + 48t - 48$$
$$\text{When } t = 0, v = -48$$
$$\text{Since } v < 0, \text{ the particle moves at first to the left.}$$
$$\text{When } v = 0, 12t^3 - 12t^2 + 48t - 48 = 0$$
$$\therefore 12(t^3 - t^2 + 4t - 4) = 0$$
$$\therefore 12(t-1)(t^2+4) = 0$$
$$\therefore t = 1$$
$$\text{When } t = 1, x = 3(1)^4 - 4(1)^3 + 24(1)^2 - 48(1)$$
$$= 3 - 4 + 24 - 48$$
$$= -25$$

The particle comes to rest at $(1, -25)$

When $t > 1, t - 1 > 0$ and $t^2 + 4 > 0$

$\therefore 12(t-1)(t^2+4) > 0$

$\therefore v > 0$

Since $v > 0$, the particle always moves to the right for $t > 1$.

9 For the first particle, $s = ut - \frac{1}{2}gt^2$ where $a = -g$

For the second particle, $s = u(t-T) - \frac{1}{2}g(t-T)^2$

The particles collide when

a i

$$ut - gt^2 = u(t-T) - \frac{1}{2}g(t-T)^2$$
$$= ut - uT - \frac{1}{2}gt^2 + gtT - \frac{1}{2}gT^2$$
$$\therefore 0 = -uT + gtT - \frac{1}{2}gT^2$$
$$= T(-u + gt - \frac{1}{2}gT)$$
$$\therefore -u + gt - \frac{1}{2}gT = 0 \qquad (T \ne 0)$$
$$\therefore gt = u + \frac{1}{2}gT$$
$$\therefore t = \frac{u}{g} + \frac{T}{2} \text{as required.}$$

ii

$$\text{When } t = \frac{u}{g} + \frac{T}{2}$$

$$
\begin{aligned}
s &= u(\frac{u}{g} + \frac{T}{2}) - g\left(\frac{u}{g} + \frac{T}{2}\right)^2 \\
&= \frac{u^2}{g} + \frac{uT}{2} - \frac{1}{2}g\left(\frac{u^2}{g^2} + \frac{uT}{g} + \frac{T^2}{4}\right) \\
&= \frac{u^2}{g} + \frac{uT}{2} - \frac{u^2}{2g} - \frac{uT}{2} - \frac{gT^2}{8} \\
&= \frac{u^2}{2g} - \frac{gT^2}{8} \\
&= \frac{4u^2 - g^2T^2}{8g}, \text{ as required.}
\end{aligned}
$$

$$
\begin{aligned}
\text{When } T = \frac{2u}{g}, s &= \frac{4u^2 - g^2\left(\frac{2u}{g}\right)^2}{8g} \\
&= \frac{4u^2 - 4u^2}{8g} \\
&= 0
\end{aligned}
$$

b This is the case when the second particle is projected upward at the instant the first particle lands. Hence there is no collision.

c If $T > \frac{2u}{g}$, the second particle is projected upward after the first particle has landed, hence no collision.